

Manipulation: Online Platforms' Inescapable Fate

Completed Research Paper

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Abstract

Online platforms are prone to abuse and manipulation from strategic parties. For example, social media and review websites suffer from the presence of opinion spam and fake reviews. Applying the economic concept of rational expectation equilibrium (REE), we explore the impact of manipulation on consumer welfare in a Twitter-like environment. We argue that the REE outcome can be decomposed into a firm-centric effect and a rational expectation effect, and the relative strength of these effects determines the final level of manipulation. We also examine the effect of competition on firms' manipulation levels. We find that the combination of a competition effect and a rational expectation effect determines the overall effect of competition on strategic manipulation. This research sheds light on the reliability of opinion mining, and contributes to our understanding of strategic manipulation in the context of sentiment analysis.

Keywords: Business value of IS, Economics of information systems, Sentiment analysis

Introduction

"You may fool all the people some of the time; you can even fool some of the people all the time; but you cannot fool all of the people all the time."

--- Abraham Lincoln (Attributed)

Much economic activity involves the understanding of consumers' preferences and the subsequent recommendations of products of interest, both of which are instrumental to product-selling firms' performances. Thanks to their popularity and ability to reach diverse demographic groups, internet platforms have established themselves as powerhouses where consumers seeking information can interact among themselves as well as with sellers, and sellers can actively identify target consumers and channel their advertisements accordingly. These online platforms include e-commerce website such as Amazon; social media sites such as Facebook, Twitter, and Foursquare; and recommendation and review websites such as Yelp, TripAdvisor, and Expedia. What these platforms have in common is the ability for consumers to voice their opinions, and for sellers to inform potential consumers of the quality of their products or services in various ways. These platforms also establish a better communication channel

between sellers and buyers. Both consumers and sellers can benefit from active participation on these platforms because they provide consumers with much more detailed information on products, while providing firms the opportunity to reach out to target consumers.

Producers on social media sites rely heavily on the word of mouth effect from a consumer's social connections to boost sales. Recommendations on these sites often come in the following two forms: you can either receive recommendations directly from your friends' endorsements, or you can be recommended products that are popular among your social connections. To facilitate a better recommendation, social media sites maintain a huge repository of social graphs, activities, and opinions, utilize machine learning techniques to predict users' preferences, and conduct sentiment analyses to discover sentiments at both the individual and aggregate levels. Specialized data analytic firms also provide sentiment analysis services to consumers and producers based on social media data.

However, popular platforms are prone to abuse and manipulation from strategic parties, and these manipulations can cause dire consequences. For example, to increase product visibility, producers might want to manipulate platform data by adding positive sentiments themselves, so the sentiment analysis results would be more favorable to their products; platforms, trying to increase visitor traffic and encourage more user participation, do not necessarily want to eradicate such producer-generated data pollution.

To better understand the effect of manipulation, we choose to base our analysis on Twitter, a popular microblogging service worldwide, and examine to what degree such strategic behaviors would impact consumer welfare. We emphasize that our goal in this paper is to provide a general framework to analyze the effect of strategic manipulation on consumer welfare. The model we construct in this paper is applicable to a variety of social networking and opinion forums, with Twitter being a motivating example of such a platform that might suffer from the manipulation issue. Therefore, we do not explicitly model some microblogging-specific functionality such as "retweet" or "following" in our current setup. We defer the discussion of manipulation specific to microblogging platforms to future studies.

The use of Twitter in marketing and advertising arenas has been remarkably dynamic, so are the speculations of the underlying business values associated with individual Twitter accounts and the huge trove of tweets. Ever since it went public, Twitter's stock price and market valuation have been extremely volatile, mostly because investors do not yet have a clear picture of how much the presence of Twitter has exactly altered the business landscape, nor has Twitter found a concrete path from tweets to profits. Even the metrics that have been used to estimate Twitter's success remain rather unsophisticated, including counting the number of monthly active users (MAUs), and calculating the growth rate of its user base. This choice of metrics implies that Twitter would encourage users to be actively engaged in conversations and to invite friends to join them, both of which could explain why it would actively or passively allow for some level of manipulation.

A severer issue related to Twitter's advertising and sentiment aggregation efficacy is the prevalence of spam tweets and robotic programs. They have both been flying under the radar but quickly undermining legitimate advertisers' efforts to connect with target consumers, and data analysts' tasks to aggregate public sentiments (Coy 2013). While the popularity of Twitter attracts advertisers to pay to increase their Twitter presence, malicious individuals or even competitors can write simple programs to achieve the same level of advertising without Twitter's spam filter and verification mechanisms detecting them. A New York Times article (Urbina 2013) lists some common goals these bots are designed to achieve: voice synthetic opinions to influence elections and stock market, or even to flirt with people—none of these matters to Twitter's business sustainability nearly as much as how these bots, which strategic firms deploy, can also be manipulated to produce massive numbers of spam tweets to sabotage legitimate advertising campaigns and contaminate sentiments. Figure 1 shows a screenshot of a website that sells Twitter accounts.

Generally speaking, manipulation is a result of lack of awareness, absence of verification mechanisms, platforms' incentive, or the nature of the chosen business model. Twitter does not have a proper verification mechanism to filter out *opinion spam*, i.e., fictitious and often fraudulent reviews that are written specifically to deceive readers (Ott et al. 2011; Ott et al. 2012; Mukherjee et al. 2013); it is especially susceptible to sentiment contamination where advertisers deliberately manipulate public

opinions by flooding the twittersphere with positive tweets and by exploiting spamming techniques to create positive word of mouth.



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 **For sale**

Provider	Quantity	Rate for 1000
Twitter.com	27729	1K-10K: \$20 10K-20K: \$18 20K+: \$17
Twitter.com UK	13782	1K-10K: \$20 10K-20K: \$18 20K+: \$17
Twitter.com HM	13236	1K-10K: \$20 10K-20K: \$18 20K+: \$17
Twitter.com+Avatar	898	1K-10K: \$25 10K-20K: \$22.5 20K+: \$20
Twitter UK +Avatar	15942	1K-10K: \$25 10K-20K: \$22.5 20K+: \$20
Twitter.com Profiled	409	1K-10K: \$30 10K-20K: \$28 20K+: \$27

Figure 1: Buy Accounts

We explore the relationship between the level of sentiment manipulation and consumers' actions under rational expectation equilibrium in the current study. Our findings suggest that, when advertisers are low quality producers, the level of manipulation will be higher than the case when advertisers are high quality producers. We also examine how the level of manipulation would be different if consumers were aware of the presence of fraudulent information, and we find that the negative effect of manipulation is the largest when consumers are naive and not aware of any manipulation. Our results reflect the combination of two effects that determine the equilibrium outcome: a *firm-centric effect* and a *rational expectation effect*. The firm-centric effect drives the firm to pursue a high level of manipulation, since more fake information would make consumers more likely to believe that a given product is of high quality; in contrast, the rational expectation effect dissuades the firm from manipulating too much, since the more manipulation, the more consumers will discount any information they receive. The relative strength of the two effects determines the total effect. We also examine the effect of competition on firm manipulation. Similarly, the equilibrium outcome can be decomposed into a *competition effect* and a *rational expectation effect*. The competition effect induces firms to manipulate more in the presence of rivals, while the rational expectation effect discourages manipulation.

Although researchers in computer science have made efforts to detect and filter out malicious behaviors, advertisers and platforms' economic incentives are more difficult to change. In addition to modeling consumers' response to the presence of spam, we also discuss how verification strategies and better control of message content can potentially curb the manipulation issue. We point out that firms' strategic spamming behaviors will stifle Twitter's attempt to profit from providing advertising opportunities for potential businesses. This implies that Twitter's business model, which hinges on both advertising and sentiment aggregation, is handicapped by its lack of verification and content monitoring. Our research contributes to the literature on sentiment aggregation by formulating a rational expectation equilibrium framework to model the firm's incentive to manipulate, and by analyzing the resulting effects on sentiment analysis and consumer welfare. Our analyses also suggest that practitioners should be cautious

when conducting sentiment analysis on user generated content, as the designs of these platforms make them susceptible to strategic manipulation.

Literature Review

User generated content (UGC) and social media data have been used in all aspects of decision making processes. In the computer science literature, O'Connor et al. (2010) connect public sentiment measured from Twitter data with public opinion surveys from polling organizations, and they find that these two data sources correlate with each other; Bollen et al. (2011) show that certain dimensions of public mood, reflected in tweets, can be used to predict stock market performance. The information systems community has also embraced the power of social media. Rui and Whinston (2011) implement a Business Intelligence (BI) system based on tweets to forecast movie box office revenues. Their results demonstrate that the forecasts of opening weekend box office revenue can be improved by incorporating Twitter data. In another paper, Rui et al. (2013) examine the Twitter word of mouth (WOM) and its implications on product sales.¹

Despite the effectiveness of UGC and social media data in improving business decisions, several studies have empirically shown the existence of widespread manipulation practices on these sites. Mayzlin et al. (2013) examine the prevalence of difficult-to-detect fake reviews on popular review websites. More specifically, they use a difference in differences approach to look at how hotel characteristics and ownership structure affect the level of review manipulation, which consists of posting positive reviews for one's own business and manufacturing negative reviews for competitors, on travel websites Expedia.com and TripAdvisor.com. Luca and Zervas (2013) investigate the presence of restaurant review fraud on another review website, Yelp.com. They find that positive review fraud is related to reputational concerns, while negative review fraud is more likely due to competitions. Anderson and Simester (2014) offer a different perspective on the nature of deceptive reviews. Using a dataset from a private apparel retailer, they find that, in addition to firms' strategic behaviors, customers without clear financial incentives to manipulate product ratings might still write reviews on products they did not purchase.

Besides review websites, social media platforms also suffer from manipulative behaviors. Stringhini et al. (2012) detail the existence of Twitter Account Markets that aim at inflating one's number of followers as well as sending out advertising tweets at a large scale. Messias et al. (2013) construct fake accounts on Twitter to demonstrate how these accounts' influence measures can be significantly improved by following simple automated strategies. The results of these studies imply that the credibility of UGC and social media data can be questionable. Therefore, decisions made based on questionable data can be harmful to the decision maker's welfare. Manipulation on these platforms also has behavioral implications. Adomavicius et al. (2013) examine the effect of recommendation on consumers' preference formation by manipulating the predictions made by recommender systems, and they find the existence of a strong anchoring effect. In other words, consumers' constructed preferences can be effectively influenced by suggestions made by recommender systems. Their findings suggest that strategic recommender systems can intentionally provide recommendations that would result in systematic biases. Muchnik et al. (2013) conduct a randomized field experiment and show that the collective intelligence among a group is often affected by social influence bias where an individual's behavior is influenced by the aggregate.

Dellarocas (2006) constructs a theoretical model on firms' manipulative behaviors. He shows that manipulations could be beneficial to consumers if firms' manipulation strategies are monotonically increasing in their true qualities. He also shows that, under certain threshold conditions, firms would actually benefit if manipulation were not possible. Mayzlin (2006) examines marketers' incentives to generate anonymous promotional messages online. Using a game theoretic model, her results show that, contrary to traditional advertising strategies, firms producing low quality products would engage in more promotional chat than those producing high quality products. This is because high quality product benefits from positive WOM which substitutes for advertising, while low quality product does not. Our

¹ Netzer et al. (2012) use network analysis and text-mining techniques to uncover market structure by analyzing UGC in both the online sedan cars and diabetes drugs forums. Hill and Benton (2014) make the connection between TV shows and UGC on Twitter, and, in aggregate, they demonstrate how one can estimate the demographics of different shows' viewers by analyzing their tweets. These estimated demographics can then be used to improve TV show and brand recommendations.

paper is distinct from these two prior studies in the following ways: (1) we formally define rational expectation equilibria in the context of sentiment analysis, and explicitly compare the differences induced by having rational-expectation-forming consumers and naive consumers; (2) Mayzlin (2006) assumes that some portions of the consumers are “informed” in the sense that they know exactly the quality of the product. In our setting, we do not require that the consumers know the product quality; instead, the informed consumers in our setup only have to know that a proportion of messages are posted strategically by the firm itself.

Distinct from previous studies, we adopt the rational expectation equilibrium framework, popularized by the influential works of Sargent and Wallace (1975) and Lucas (1976), to examine firms’ incentive to generate opinion spam in a Twitter-like social broadcasting environment. The use of a rational expectation equilibrium in modeling both firms and consumers’ equilibrium behaviors can also be found in Su and Zhang (2009).

Model

We choose to base our analysis on a Twitter-like environment, and therefore we use the word “Twitter” to refer to opinion platforms, and “tweets” to refer to the content on these platforms. We emphasize that our analysis is not limited to Twitter; instead, the theoretical results can be applied to any opinion platform or review website that features user generated content.

Baseline Model (No Manipulation)

Formally speaking, we assume there are n consumers who are interested in N products, with unknown qualities, q_1, \dots, q_N , sold by N different firms. Without loss of generality, we focus our discussion on the case where $N = 1$, and delay our discussion on multiple firms to a later section. These consumers arrive sequentially, and the quality of any given product can be either good, G , or bad, B , i.e. $q \in \{G, B\}$. Consumers receive a private signal, s , independently and identically drawn from a Bernoulli distribution, p , and this signal can be either high, H , or low, L , with the high (low) signal meaning the product is more (less) likely to be of good quality. We assume a common prior on the probability that the product is of good quality: $\Pr(G) = \Pr(B) = 1/2$. This means that, without seeing any signals, the product is equally likely to be of good or bad quality. We further assume that signals are informative with the following values: $\Pr(H | G) = \Pr(L | B) = 3/4$, and $\Pr(L | G) = \Pr(H | B) = 1/4$. Applying the law of total probability, we can derive the prior probability of receiving an H or an L signal: $\Pr(H) = \Pr(H | G)\Pr(G) + \Pr(H | B)\Pr(B) = 1/2$, and, similarly, $\Pr(L) = 1/2$. Before making the purchasing decision, the consumer would log on to Twitter to tweet her signal, and then search for tweets regarding this specific product. Based on her own signal and other people’s tweets, she uses Bayesian update to calculate the posterior probability of the product quality. Suppose she receives a H signal. Applying the Bayes rule, the posterior probability of the product being of good quality is $\Pr(G | H) = 3/4$. More generally, we assume that she sees a total of n H signals and m L signals, inclusive of her own signal, and also assume that signals are independent draws. The consumer makes her decision based on the average sentiment conveyed by these tweets, together with her own signal. Normalizing the utility of purchasing a product of good quality, $U(G)$, to be 1, and $U(B) = -1$ for bad quality product, the consumer’s expected utility of purchasing this product can be expressed as $E[U | \text{signals}] = q \cdot U(G) + (1 - q) \cdot U(B)$, where we define $q = \Pr(G | \text{signals})$ to be the posterior probability of the product having good quality, after inspecting the tweets and her own signal. The following proposition characterizes the condition under which the consumer will purchase the product.

Proposition 1 (Consumer’s Decision Rule) *Let n and m be the number of H and L tweets, respectively, that a consumer observes on Twitter in the case where there is no firm manipulation. Then the consumer will purchase the product if $m < n$. (Proof available upon request.)*

The intuition behind this proposition is that, since there is no firm manipulation and all tweets are genuine, if the number of high signals is larger than that of low signals, it is more likely that the product is of good quality. Therefore, the consumer will purchase the product if $m < n$. The firm, in contrast, shares common priors with consumers. To figure out the demand and thus the revenue level, the firm would estimate the number of consumers that will purchase its product by inspecting the prior on both the signal distribution and the number of consumers who are to make purchasing decisions. Similar to the earlier discussion on the consumer's purchasing decision, we can model the sequence of tweets, and thus the signals, each of the consumers sees when she searches for this product on Twitter, as a sequence of H 's and L 's. Since the order of signals in a sequence does not affect the consumer's decision (with the exception of her own signal which works as a tie-breaker), we can instead model these signals as a combinations of H 's and L 's. This combination of signals can be represented as follows. For the i -th consumer, let s_1, \dots, s_{i-1} be the signals she sees. Also, let H take the numerical value of 1 and let L be -1 . Then the i -th consumer's purchasing decision, denoted as D_i , can be modeled as the process of first summing up the numerical values of H 's and L 's in the combination of signals she sees, i.e. $\sum_{k=1}^{i-1} s_k$, and then inspecting her own signal, s_i , to check if the sum is greater than the threshold value, 0. Then, assuming a more general case where the firm has a prior belief p on how likely a consumer would receive an H signal, s_i follows a Bernoulli distribution: $s_i \sim \text{Bernoulli}(p)$. In the following discussion we suppress the functional dependence on the prior belief p for notational ease. Notice that her decision, D_i , can be expressed as

$$D_i = \begin{cases} 1 & \text{if } \left\{ \sum_{j=1}^i s_j > 0 \right\} \text{ or } \left\{ \sum_{j=1}^i s_j = 0 \text{ and } s_i = 1 \right\}, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

From the firm's perspective, the expected value of consumer i 's decision, $E[D_i | p]$, can be understood as the expected number of consumers that will purchase this product. This expectation can be derived from the following equation:

$$E[D_i | p] = \Pr(\{D_i = 1\}) = \Pr\left(\left\{ \sum_{j=1}^i s_j > 0 \right\}\right) + \Pr\left(\left\{ \sum_{j=1}^i s_j = 0 \right\} \wedge \{s_i = 1\}\right). \quad (2)$$

Finally, assuming there are n consumers to make purchase decisions, we can calculate the firm's expected profit by summing over all consumers' expected values of decisions, with the firm having a prior belief p and the price of the product normalized to 1: Expected Profit = $E[\pi] = \sum_{i=1}^n E[D_i | p]$, where each $E[D_i | p]$ is obtained via equation (2), and where we treat the product's production cost as a sunk cost, and therefore exclude it from the profit equation.

Manipulation Model

Naïve Consumers

Knowing consumers' decision rules, the firm would have a strong incentive to post positive tweets online, masqueraded as any normal consumer's signal-reporting tweet, in order to attract more consumers. This form of opinion spam could be used for advertising, political propaganda, and many other purposes. Consumers might make suboptimal choices if such manipulation behaviors, and the subsequent sentiment pollution, remain unknown to them. In this section, we explore the effect of manipulative tweets on Twitter by quantifying the optimal level of manipulation in which the firm would engage,

together with the corresponding profit level, assuming consumers are unaware of the presence of opinion spam and thus believing in the tweets completely.

Similar to the setup in the previous section, we assume there are n consumers interested in a firm's product, and the firm has access to common priors, and can form a probabilistic belief, p , on the probability that any given consumer would receive an H signal, $\Pr(H)$. We assume that the number of interested consumers, n , is known to the firm ex ante, and that the firm has already produced the products, so the production costs are sunk and not included in the firm's profit maximization problem. Therefore, the only decision the firm has to make is to decide the number of fake positive tweets to post. We choose to call these tweets "fake" because in our setup tweets are assumed to convey the real signals consumers received, whereas these tweets posted by the firm are purposely produced to manipulate consumers' purchasing decisions.

To simplify the problem, we assume the firms can only post fake tweets at the beginning of each advertising campaign. Once it posted fake tweets it would leave the platform and collect revenue from consumers' purchases. In order to determine the number of fake positive tweets to be posted, the firm firstly inspects the consumer's decision rule and the expected demand level. Following the decision rule given in equation (1), the firm knows that a consumer will purchase the product as long as the number of positive tweets is greater than that of negative tweets. Therefore, a plausible strategy is to make sure that, for every consumer, positive tweets outnumber negative tweets by at least one. In other words, by posting $n + 1$ fake positive tweets, the firm can ensure that consumers will all purchase, because even in the worst case scenario where everyone receives an L signal, the total number of positive tweets, $n + 1$, still exceeds that of negative tweets, n . This obviously depends on the firm's cost of posting fake tweets. Here we assume that the cost of manipulating can be expressed as a general cost function, $a(m)^b$, where m is the number of fake positive tweets that the firm chooses to generate, and both a and b are some known constants. Then, extending the baseline model, the firm's optimal manipulation level can be derived by maximizing its profit function:

$$m^* = \arg \max_m \sum_{i=1}^n E[D_i | p, m] - a(m)^b. \quad (3)$$

Rational Expectation Model with Rational Consumers

A more interesting case is when some consumers realize that the firm is manipulating its Twitter sentiment, and therefore discount the proportion of positive sentiments to correct for the potential upward bias caused by these manipulative positive tweets. We extend our baseline model to accommodate for both (A) the firm's manipulative behaviors, and (B) consumers' rationally discounting of any positive tweets. More specifically, using the concept of rational expectation equilibrium, at the equilibrium, the firm will decide on a level of manipulation, i.e. the number of fake positive tweets, to post on Twitter, and consumers will rationally expect this level of manipulation and discount the positive tweets they see on Twitter.

Formally speaking, let m be the number of fake positive tweets that the firm decides to post on Twitter. We again assume that the firm posts all of its tweets before any consumer tweets, and consumers arrive sequentially. Then for the i -th consumer, the combination of tweets she would see on Twitter includes both previous consumers' tweets, $s_1 \dots s_{i-1}$, her own tweet, s_i , together with the firm's m fake positive messages. The consumer's decision rule is the same as the baseline model where she would compare the number of positive tweets with that of the negative tweets. The difference here is that we assume the consumer is aware of the possibility that certain tweets might have been manipulated, but she is not able to distinguish genuine positive tweets from fake positive tweets, presumably because the firm uses different aliases to post. Therefore, a rational consumer would discount the number of positive tweets by some discount factor, f , which corresponds to her belief of the level of genuine tweets among all positive tweets. Since we only consider rational expectation equilibria, this discount factor f must also equal the proportion of genuine positive tweets among all positive tweets posted on Twitter. Based on its belief of

the probability of any consumer receiving an H signal, the firm's problem is to choose the number of fake positive tweets to post. Let the firm's belief of any consumer receiving an H signal be some probability p , and let the total number of consumers be n . Then the expected number of genuine positive tweets is $n \cdot p$. Depending on the value of m , the expected proportion of genuine positive tweets and, equivalently, the consumer's discount factor, f , can be expressed as $f = (n \cdot p) / (n \cdot p + m)$. For simplicity, we assume the consumer only discounts positive tweets she sees, while she trusts her private information and thus never discounts her own signal. Also, let h_i denote the quantity $\sum_{j=1}^i \mathbb{I}(s_j = 1)$, where $\mathbb{I}(\cdot)$ is the indicator function. Notice that in this case, aside from her private signal, the i -th consumer sees $(m + h_{i-1})$ positive tweets and cannot distinguish the firm's fake positive tweets from genuine positive tweets. At a rational equilibrium, she rationally and correctly expects the proportion of genuine tweets, and discounts positive tweets by f . So the "effective" level of positive tweets is now

$$f \cdot (m + h_{i-1}) \cdot 1 = \frac{n \cdot p}{n \cdot p + m} \cdot (m + h_{i-1}), \quad (4)$$

where we have first converted the H signals to 1, then discounted them each from 1 to f ; $0 \leq f \leq 1$; h_{i-1} follows a binomial distribution. The consumer's decision rule is now

$$D_i = \begin{cases} 1 & \text{if } f \cdot (m + h_{i-1}) + [(i-1) - h_{i-1}] \cdot (-1) + s_i > 0, \\ 1 & \text{if } \{f \cdot (m + h_{i-1}) + [(i-1) - h_{i-1}] \cdot (-1) + s_i = 0\} \wedge \{s_i = 1\}, \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Denoting the quantity $[f \cdot (m + h_{i-1}) + [(i-1) - h_{i-1}] \cdot (-1)]$ as S_i^* and following the derivation in the baseline model, we have

$$E[D_i | p] = \Pr(\{D_i = 1\}) = \Pr(\{S_i^* + s_i > 0\}) + \Pr(\{S_i^* + s_i = 0\} \wedge \{s_i = 1\}) \quad (6)$$

Let π denote the firm's profit level. Then the firm's profit maximization problem is

$$\max_m \pi(p, m(p)) = \max_m \sum_{i=1}^n E[D_i | p] - a(m)^b, \quad (7)$$

and the optimal level of manipulation, m^* , is given by $m^* = \operatorname{argmax}_m \sum_{i=1}^n E[D_i | p] - a(m)^b$.

To summarize, we provide a formal definition of a rational expectation equilibrium involving one strategic firm and n rational consumers.

Definition 1 A rational expectation equilibrium with one strategic firm and n rational consumers consists of $(f, \{D_i\}_{i=1, \dots, n}, m^*)$ which satisfies

$$\begin{aligned} (i) \quad & D_i = \mathbb{I}(S_i^* + s_i > 0) + \mathbb{I}(\{S_i^* + s_i = 0\} \wedge \{s_i = 1\}), \\ (ii) \quad & m^* = \operatorname{argmax}_m \pi(p, m(p)) = \operatorname{argmax}_m \sum_{i=1}^n E[D_i | p] - a(m)^b, \end{aligned} \quad (8)$$

$$(iii) f = \frac{n \cdot p}{n \cdot p + m^*}, \text{ where } S_i^* \text{ is defined as } [f \cdot (m + h_{i-1}) + [(i-1) - h_{i-1}] \cdot (-1)] .$$

Notice that condition (i) is from the rational consumer's optimal decision D_i based on her belief of the proportion of genuine tweets, f ; condition (ii) is from the strategic firm's optimal choice of its manipulation level; condition (iii) is the consistency condition.

Results

In this section we describe our simulation results and compare how the rational expectation equilibrium model provides different estimates from the model of naive consumers. For the first set of simulations, we set the number of consumers to be 30; the cost function, $a(m)^b$, is specified with $a = 0.1$ and $b = 2$, i.e. a convex (quadratic) cost function. The intuition behind using the above functional form to specify the manipulation cost is as follows. We use this cost function to model the cost of posting fake messages, as well as reflecting how strict the verification mechanism is on the given platform. The more strict the verification mechanism is, the more costly it should be for firms to manipulate successfully. Therefore, we believe that a convex cost function would be suitable for our simulations. Several past studies, including Mayzlin (2006), also use a convex function to model the cost of manipulation. We note that our simulation results are robust to different values of a and the number of consumers. This cost function specification implies that the marginal cost of producing a fake tweet increases in the manipulation level, potentially because it would become increasingly difficult to avoid detection once the number of fake tweets is large, and thus more costly for the firm to manipulate. We vary the probability that the signal associated with this product is high from 0.3 to 0.7. Notice that this probability is positively associated with the product quality, with good quality product having a higher probability of its signals being high. The manipulation results based on different probability levels allow us to understand how product quality affects the firm's incentive to manipulate. Figure 2(A) shows different levels of manipulation as we vary the probability of the product being of good quality from 0.3 to 0.7. We can see that, in general, the higher the probability of the product being of good quality, the less manipulation the firm would generate. This result is consistent with the findings from Mayzlin (2006) where she shows that firms that produce low quality products spend more on promotional chats. It is worth noting that there is a significant difference between the naive and the rational cases where the firm would try to manipulate more when users are naive.

To calculate consumer welfare, we assume the utility of buying a good product to be 1, that of a bad product to be -1 , and the utility of not buying any product to be 0. We also assume that consumers are risk neutral. Figure 2(B) plots the levels of consumer welfare for both the naive and rational cases, against varying levels of product quality. This figure shows that the difference in welfare level between naive and rational consumers is the largest when product quality is low. In other words, naive consumers suffer the most when facing low quality products, while there is no significant difference between naive and rational consumers when product quality is sufficiently high. On the other hand, firms in general would benefit more when consumers are naive, and the difference is most pronounced when product quality is low, as shown in Figure 2(C). This means that, as long as consumers are naive, firms do not necessarily need to increase the quality of their products; instead, they could potentially rely on posting fake positive tweets in order to attract consumers.

We also simulate the case where the cost function is linear instead of convex. The results are similar to the convex cost case, except for the manipulation level when consumers are rational. We can see from Figure 2(D) that, given the probability of good product, the firm seems to be manipulating much more in the linear cost case compared with that in the convex cost case. In other words, when the marginal cost of manipulation is increasing, the firm manipulates more in the naive case; with a constant marginal cost of manipulation, the firm manipulates more in the rational case. This might be because that, since the cost of manipulating is relatively small, the firm would try to manipulate more when facing rational consumers in order to compensate for the discounting of positive tweets that those consumers would engage in, so the resulting positive sentiment could still be favorable for the firm. Note that the type of marginal cost faced by manipulative firms can be determined by the verification and detection mechanisms the platform

employs. For example, platforms without strict verification policies such as Twitter and TripAdvisor might fall in the category of constant marginal cost, which means that, in the rational consumer setting, firms would engage in more manipulative behaviors on these platforms. In contrast, other platforms have mechanisms that can better ensure the integrity of user generated content: Yelp has a review filter that hides suspicious reviews (Luca and Zervas 2013); only the customers who have reserved and visited restaurants are allowed to submit ratings on the restaurant reservation platform OpenTable. This type of platforms uses verification mechanisms to deter abuse and manipulation, which can be modeled as having an increasing marginal cost, and results shown in Figure 2(A), Figure 2(B), and Figure 2(C) would apply. It is worth noting that, without taking into account the consumer's forming rational expectation with regard to firms' manipulation, we would have underestimated the level of manipulation, had the cost function been linear.

To provide some intuitions for these results, we consider two forces that affect the equilibrium outcomes. Firstly, there is a *firm-centric effect* which encourages the firm to manipulate more when the consumers are rational. This is because rational consumers expect some level of manipulation, so the firm would have to exert extra efforts in order to counter the consumer's discounting; when consumers are naive, the firm can more easily convince them to purchase its product, and hence they would not have to manipulate as much. However, there is another effect, which we coined the *rational expectation effect*, that induces consumers to engage in more sentiment discounting the more likely the firm wants to manipulate, and thus the less effective the manipulation would be. Therefore, when the cost of manipulation is high, the rational expectation effect will predict a decrease in manipulation. Overall speaking, a rational expectation equilibrium reflects both effects, and the results depend on the relative strengths of them. In the convex cost case, since manipulation is relatively costly, the rational expectation effect outweighs the firm-centric effect, so we observe a lower level of manipulation in the presence of rational consumers compared with the naive setting, as illustrated in Figure 2(A). By similar reasoning, since the cost of manipulation is relatively cheap in the linear cost case, the firm-centric effect dominates, and we observe a higher level of manipulation in the rational setting than that in the naive setting, as shown in Figure 2(D). It is worth noting that, while informative, our rational expectation equilibrium results are theoretical in nature. The analysis conducted here regarding rational consumers and naive consumers are likely to differ from the real world situation, because in reality some consumers are likely to be rational while others might be unaware of the existence of any strategic manipulation. Our contribution is to provide bounds for the real situation, which lies somewhere in between the completely rational and the completely naive cases.

Multiple Firms with Rational Consumers

We can extend our discussion of rational expectation equilibrium to the case where there are multiple strategic firms and n rational consumers. We discuss the two-firm case in this paper, since adding more firms would be a straightforward generalization of the two-firm results. Before formally defining the rational expectation equilibrium, we first set up consumers and firms' decision problems as follows.

First we suppose that all players in this model possess common priors on the probability of product A receiving an H signal, denoted as p_A ; and, similarly, the probability of product B receiving an H signal, p_B . Without loss of generality, we assume $p_A > p_B$. Firm A and firm B sell similar products and the consumer can only purchase at most one firm's product. We further assume that both firms know there are a total of n consumers in the market. Similar to the one-firm case, since consumers rely on Twitter sentiment for decision making, firms have incentives to strategically post fake positive tweets in order to attract consumers to purchase their products. In a rational expectation equilibrium, the consumer does not know the actual level of manipulations firm A and firm B would pursue. Instead, she possesses some beliefs over the proportion of positively manipulative tweets among all tweets related to both firms, denoted as f^A and f^B , respectively. We assume that all consumers share the same f^A and f^B . Firm A decides the number of fake tweets it will post, m^A , given its beliefs of firm B 's manipulation level, R^B . Similarly, firm B posts m^B fake positive tweets to promote its own product, based on its belief of firm A 's manipulation level, R^A . To simplify the model, we assume that firms only

post fake positive tweets about their own products, and do not post fake negative tweets about their competitor's product.

A rational expectation equilibrium in this context means that all consumers choose a purchase strategy, $\{D_i\}_{i=1,\dots,n}$, to maximize their expected utilities, while firms maximize their expected profit, π^A and π^B , given the consumer's purchasing strategy and competitors' manipulation level. For the i -th consumer, we assume that she observes a total of k^A tweets on product A , i.e. $\sigma_1^A, \dots, \sigma_{k^A}^A$, and k^B tweets on product B , i.e. $\sigma_1^B, \dots, \sigma_{k^B}^B$, both excluding her own signals, s_i^A and s_i^B . Since she cannot distinguish genuine tweets from fake tweets, we use $\phi^A(i)$ to express product A 's discounted average sentiment that she can observe, where $\phi^A(i)$ is defined as

$$\phi^A(i) = \left[f^A \cdot \left(\sum_{j=1}^k \mathbb{I}(\sigma_j^A = 1) \right) + \left[\sum_{j=1}^k \mathbb{I}(\sigma_j^A = -1) \right] \cdot (-1) \right] + s_i^A. \quad (9)$$

Product B 's discounted average sentiment observed by consumer i , $\phi^B(i)$, can be defined similarly. Firm A 's objective is to maximize its expected profit by choosing a manipulation level, m^A , given its belief of firm B 's manipulation strategy, R^B . Its maximization problem is formulated as

$$m^A(R^B) = \operatorname{argmax}_{m^A} \pi^A(p_A, p_B, R^B, m^A); \quad (10)$$

firm B 's maximization problem is formulated similarly. Based on the single firm case equilibrium definition in equation (8), we have the following rational expectation equilibrium definition:

Definition 2 A rational expectation equilibrium for two strategic firms and n rational consumers consists of $(m^A, m^B, \{D_i\}_{i=1,\dots,n}, f^A, f^B, R^A, R^B)$ which satisfies

$$\begin{aligned} (i) \quad & D_i(f^A, f^B) = \Psi(\phi^A(i), \phi^B(i)), \\ (ii) \quad & m^A(R^B) = \operatorname{argmax}_{m^A} \sum_{i=1}^n \mathbb{E}[\mathbb{I}(D_i = A) | p_A, p_B, R^B, m^A] - a(m)^b, \\ (iii) \quad & m^B(R^A) = \operatorname{argmax}_{m^B} \sum_{i=1}^n \mathbb{E}[\mathbb{I}(D_i = B) | p_A, p_B, R^A, m^B] - a(m)^b, \\ (iv) \quad & m^A = R^A = \frac{n \cdot p_A \cdot (1 - f_A)}{f_A}; m^B = R^B = \frac{n \cdot p_B \cdot (1 - f^B)}{f^B}, \end{aligned} \quad (11)$$

where p_A and p_B are the common priors for firms' products, respectively; $\Psi(\phi^A(i), \phi^B(i))$ is defined as

$$\Psi(\phi^A(i), \phi^B(i)) = A \text{ if } \begin{cases} (\phi^A(i) > 0) \wedge (\phi^A(i) > \phi^B(i)) & , \text{ or} \\ (\phi^A(i) = 0) \wedge (s_i^A = 1) \wedge (\phi^A(i) > \phi^B(i)) & , \text{ or} \\ (\phi^A(i) = \phi^B(i) > 0) \wedge (s_i^A > s_i^B) & , \text{ or} \\ (\phi^A(i) = 0) \wedge (\phi^B(i) = 0) \wedge (s_i^A > s_i^B) \end{cases} \quad (12)$$

$$\Psi(\phi^A(i), \phi^B(i)) = B \text{ if } \begin{cases} (\phi^B(i) > 0) \wedge (\phi^B(i) > \phi^A(i)) & , \text{or} \\ (\phi^B(i) = 0) \wedge (s_i^B = 1) \wedge (\phi^B(i) > \phi^A(i)) & , \text{or} \\ (\phi^B(i) = \phi^A(i) > 0) \wedge (s_i^B > s_i^A) & , \text{or} \\ (\phi^B(i) = 0) \wedge (\phi^A(i) = 0) \wedge (s_i^B > s_i^A); \end{cases}$$

$$\Psi(\phi^A(i), \phi^B(i)) \sim \text{Bernoulli}\left(\frac{1}{2}\right) \text{ if } \begin{cases} (\phi^A(i) = \phi^B(i) > 0) & , \text{or} \\ (\phi^A(i) = \phi^B(i) = 0) \wedge (s_i^A = s_i^B = 1) \end{cases}$$

$$\Psi(\phi^A(i), \phi^B(i)) = 0, \text{ otherwise.}$$

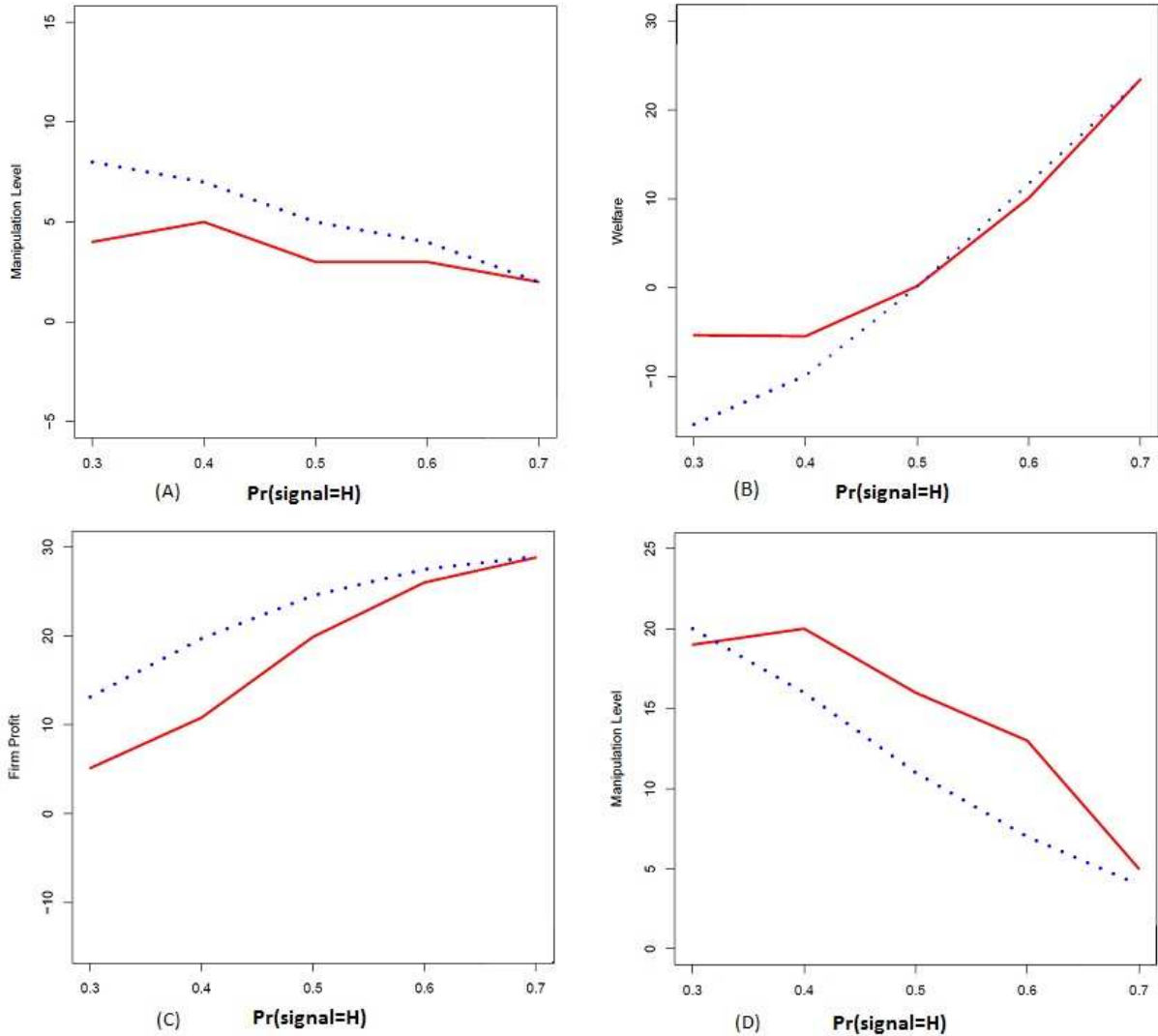


Figure 2 (A) Manipulation, convex cost; (B) Welfare, convex cost; (C) Profit, convex cost; (D) Manipulation, linear cost. Solid line: Rational consumer, Dashed line: Naïve consumer

Notice that in Definition 2, condition (i) specifies consumers' decision rules; condition (ii) and (iii) ensure that the level of manipulation that firm A and firm B attempt is equal to their respective profit-maximizing levels; condition (iv) ensures the rational expectation equilibrium in the sense that the resulting manipulation level of each firm is equivalent to its competitor's expectation and the consumer's expectation of its manipulation level. Equation (12) details the consumer's decision rule where this rule makes use of the following principles: (1) the consumer would always choose the product with a higher discounted tweet sentiment, as long as that sentiment is positive. If both products' sentiment levels are negative, then the consumer will not purchase either one; (2) the consumer's own private signal is assumed to be always genuine, and serves as a tie breaker when the number of discounted positive tweets equals the number of negative tweets; (3) if a tie persists after applying the previous two principles, then we assume that the consumer will take a Bernoulli(1/2) random draw to decide which product to purchase.

Results

Figure 3(A) and 3(B) plot the simulation results from a rational expectation equilibrium with two competing firms. Each plot represents three sets of firm types, indexed by "High", "Medium", and "Low". The "High" case is specified with the two firms both having high probabilities of producing good quality products, with the two probabilities being $\Pr(\text{signal} = H) = 0.7$ and 0.6 , respectively. Intuitively, we can understand these probabilities as different firm types, with the higher probability firm producing a better quality product, and the lower probability firm producing a lower quality product. Figure 3(D) lists all combinations of firm types that are used in our simulation. The results in Figure 3(A) correspond to the profit, sales, and manipulation levels of Firm A when it competes with Firm B, with the associated firm product qualities listed in Figure 3(D). Note that the focal firm in Figure 3(A) is Firm A, namely the firm producing the higher quality product among the two competing firms; Figure 3(B) illustrates the effect of competition on Firm B, the firm producing the inferior product among the two competing firms. We also point out that the effect of competition is measured by comparing the level of manipulation, consumer welfare, firm sales, and firm profits, in the two-firm case, with those in the monopoly setting, holding the focal firm's product quality constant. For example, the "High" case in Figure 3(A) represents the effect of competition on a firm with product quality 0.7 when facing a competitor that produces a product of a lower quality, 0.6. As mentioned earlier, we simulate both the case where this high quality firm faces no competition, and the case where it faces competition from a lower quality firm, and compare these two cases to estimate the effect of competition. Similarly, the "High" case in Figure 3(B) illustrates the effect of competition on a firm which produces a product with a quality level of 0.6, when competing with a firm that produces a better product with a quality level of 0.7.

To see how the cost structure of manipulation changes the effect of competition, we also simulate the rational expectation equilibrium across three quality levels where the cost to manipulate is linear in manipulation, shown in Figure 3(C). Comparing Figure 3(A) and Figure 3(C), we can see that the changes in the manipulation level facing competition are much larger in the linear cost case than those in the convex cost case. This is because it costs less for the firm to manipulate when the cost is linear than when it is convex. It is worth noting that the changes in profit and sales levels are also larger in the linear cost case than those in the convex cost case, most likely because the feasible range of manipulation in the linear cost is much larger than that in the convex cost.

Comparing the effects of competition on both Firm A and Firm B across these three test groups informs us of how firms of different quality levels are affected by competition in a manipulative environment. Regardless of the firm being better or worse than its competitor, competitions drive down the profit and sales levels across all three groups, as can be seen by comparing Figure 3(A) and Figure 3(B). In particular, the worse firm in a pair of competing firms suffers more from the competition, both in terms of profits and sales. Interestingly, when facing competition, a high quality firm tends to increase its level of manipulation much more in the case when its quality is already better than its competitor, than the case when its quality is worse than its competitor, as can be seen by comparing the "High" case in both figures. On the other hand, a low quality firm tends to decrease its level of manipulation much more in the case when its quality is better than its competitor, than the case when its quality is worse than its competitor, as can be seen by comparing the "Low" case in Figure 3(A) and 3(B). It is also worth noting that the firms

in both the “Medium” and “Low” groups suffer less from the competition, most likely because their profit and sales levels are lower to start with, even without any competition.

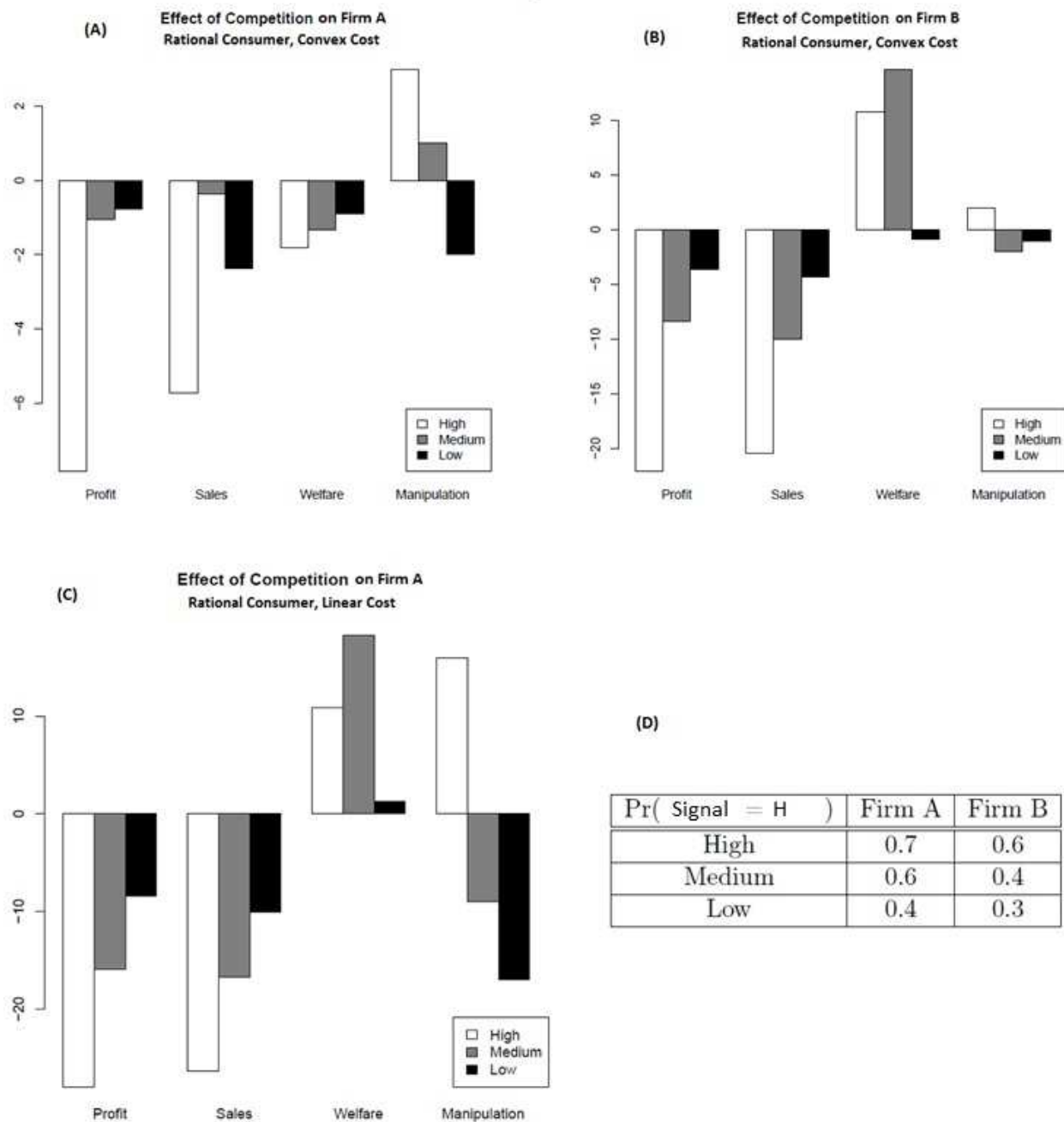


Figure 3: (A): Effect of Competition on Firm A, convex cost; (B) Effect of Competition on Firm B, convex cost; (C) Effect of Competition on Firm A, linear cost; (D) List of Cases

This decrease of manipulation level for the low quality firm is surprising because, intuitively speaking, firms would want to promote their own products more when a competitor enters the market. For

example, Mayzlin et al. (2013) show that fake reviews are more prevalent for hotels that face a strong competition. The intuition behind our surprising results is as follows. Similar to the one-firm case, we can decompose the overall effect of competition into the following two effects: the *competition effect* and the *rational expectation effect*. The competition effect induces the firm to engage in more manipulation in order to attract consumers to purchase its product instead of the competitor's product. On the other hand, the firm also cares about the rational expectation effect in the sense that the more manipulation, the more discounting the consumers will apply. The resulting equilibrium outcome can be seen as a combination of these two effects. If the competition effect outweighs the rational expectation effect, then the firm will manipulate more, and vice versa. This explains the decrease of manipulation for low quality firms: if the firm's product quality is already low, then it would probably be detrimental to induce more discounting from consumers, and hence the firm would want to engage in less manipulation when its product quality is low.

We also explore the effects of competition on consumer welfare in each of the groups by comparing the welfare levels in the two-firm environment with those in the one-firm environment. The welfare changes in the presence of competition are also shown in Figure 3(A) and Figure 3(B). From these figures we can see that, when a firm is joined by a competitor with a higher quality, the consumer welfare level drops, regardless of the quality of these firms. On the other hand, when a firm is joined by a competitor with a lower quality, the consumer welfare level increases significantly when the quality of these firms are not too low. It might seem contradictory to the intuitive speculation that competition should lead to an increase in consumer welfare. This is because we assume that firms do not engage in a price-setting competition; instead, they only rely on manipulating sentiments on Twitter to compete with each other. Therefore, consumers will not benefit from any price reduction that would have happened in other competitive situations. This assumption makes sense in settings where the price variations among different producers are small, such as the movie industry, and with this assumption we are able to focus our discussion on the effect of manipulation alone. Similar to the results in the single-firm case, the multiple-firm rational expectation equilibrium serves as a starting point for us to understand the consequence of firm manipulation when the consumers are aware of such strategic behaviors, while in reality there will be some consumers that are more rational than others. We will examine the case where the consumers are of different types in future research.

Conclusion

In this paper we studied the effect of manipulation on consumer welfare, and the effect of competition on the firm's manipulation decision, both in the rational expectation equilibrium framework. We emphasized the importance of recognizing the existence of strategic manipulation, because researchers as well as practitioners have been collecting and analyzing tremendous amount of social and opinion platform data to conduct sentiment analysis, often without explicitly adjusting for fake sentiments. Our results suggested that the equilibrium outcomes of manipulation level can be decomposed into a firm-centric effect and a rational expectation effect. When marginal cost to manipulate is increasing, the rational expectation effect dominates the firm-centric effect, and the firm will consequently manipulate less. We also considered the effect of competition on the firm's incentive to manipulate. We found that, when the firm's product quality is low, it is likely that the rational expectation effect will dominate the competition effect, which would discourage the firm from manipulating.

We recognize several limitations in the current research: (1) we only considered fake positive messages in our theoretical model. However, in reality, firms often post negative messages about competitors to lower their competitors' sentiment; (2) Our current results were obtained from numerical simulations. We plan to derive analytical results in our future research, and to decompose the total effect into the firm-centric and rational expectation effects analytically; (3) We were unable to empirically estimate the relative proportions of rational and naive consumers on the platforms. As a future extension, we plan to incorporate the possibility of posting negative messages, and to conduct an empirical study on manipulative behaviors on opinion platforms. Moving forward, Twitter, as well as other review platforms, must address the spamming and verification issues in order to avoid the danger of losing values and relevance in the years to come. Other online platforms should also reassess their susceptibility to manipulative behaviors, and find ways to maintain their credibility for a sustainable development.

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